THE UNIVERSITY OF ALBERTA

A GENERAL CLOSED COMPETITIVE BIDDING MODEL

BY

© LARRY I. DAWSON

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UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled A GENERAL CLOSED COMPETITIVE BIDDING MODEL submitted by Larry I. Dawson in partial fulfilment of the requirements for the degree of Master of Business Administration.

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ABSTRACT

Closed competitive bidding is generally accepted as an efficient means of allocating scarce resourses within the economy and appears to be gaining widespread use, partly due to governments' increased activity in the business sector. However, bidding poses some formidable problems to competing companies' managements due to the uncertainties and complexities that may exist when following this method.

This paper structures the closed competitive bidding environment by means of a model. Then, given a set of bidding information and with the aid of this model, a company's management may be able to more profitably allocate the resources that they have at their disposal.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

In general, a competitive situation is one in which two or more parties are in conflict relative to a set of their objectives. These parties "co-operate" relative to either an objective they share in common, or an objective of a third party served by the competitors. One type of competitive situation is one in which bidding takes place: bidding for contracts, concessions, rights and licenses. In fact, pricing of products and services can be conceived as bidding for customers' dollars.

A bid is an offer of a price (to be received or given) in an attempt to secure an item of value, a prize. The winning bid, as dictated by the rules and translated by the judge(s), is generally the lowest bid announced and accepted in anticipation of performing a service, or the highest bid offered and accepted in the expectation of obtaining a privilege. Generally, it is a bidding contract if a bid is accepted and is legally enforceable in the courts.

Scope of the Research

The simplest bidding procedure to analyze is that of the ordinary or progressive auction. Bids are freely made and publicly announced until no other potential purchaser wishes to make any further lower bid. (We are assuming here, and will continue to do so for the remainder of this paper, that the winning bid is the lowest bid submitted). The anticipated result among "rational" competitors is for the prize to be awarded to that competitor who places the lowest valuation on the prize and at a price of the first increment below the second lowest valuation. For example, if there are two competitors and one competitor places a valuation of $x on the prize, then the other competitor should be able to receive the prize for $x-.01 if they place a valuation of less than $x on the prize and the minimum bid increment is $.01.

Before any type of bidding actually commences, there are two major areas of uncertainty existing:

1. the valuation of the prize; and
2. the competitive environment.

In auction type bidding, the competitive environment becomes known as the bids are made, announced and re-made. This leaves the valuation of the prize as the only major source

of uncertainty.

Contrast this bidding situation with the type that is discussed in this paper, close competitive bidding. Now only one bid may be made and all bids must be submitted by some specified time. If there is perfect information, this bidding situation would be analogous to auction bidding. But as perfect information generally does not exist regarding either the valuation of the prize or, in this situation, the competitive environment, an analysis of closed competitive bidding becomes a formidable task.

But, since the essence of managerial responsibility is the balancing of payoff and risk in light of future uncertainties, and the making of "good" decisions based on the outcome of this complex process, some attempt should be made to model the bidding situation even though it may be only tentative. The scope of this paper is to structure the closed competitive bidding situation by means of a model, and describe the payoffs and their associated risks.

Research Methodology

This paper presents the results of normative research on closed competitive bidding. That is, the objective is not to discover how a competitor acts in a

given bidding situation, but rather how they should act if they desire to achieve certain goals. However, the actual behavior of competitors should be described because:

a. Some knowledge of how competitors do, in fact, behave is helpful in identifying the problem.

b. The behavior of other competitors constitutes a part of the environment in which a given company operates. Hence, a description of their behavior is relevant to this company's own decision problem.

c. It is of interest to compare the prescriptions for behavior arrived at here with the actual behavior of competitors.

This last point, the testing of the model we derive, is not attempted in this study but offers some challenging possibilities for further research.

The principal technique we employ in this paper is one of model-building. A model is an abstract representation of a part of the real world. In building a model, the relevant features of the real world are "mapped" into a system with an analogous structure; or, to put it in another way, the elements of the system bear the same relationships to each other as do the features of the real

world. Consequently, by manipulating the system, we can predict what would happen if the corresponding real world features are changed.

Specifically, if the real world bidding situation can be described by an appropriate model, then the effects that different bidding strategies will have on some appropriate criteria can be predicted by manipulating the model. This then assists in the determination of a bidding strategy that is optimum, or near optimum.

Outline of the Research

In Chapter II, the relevant published works on closed competitive bidding are reviewed. A conclusion from this examination is the apparent lack of creditable material in this area.

In Chapter III, we develop a closed competitive bidding model. A bidding contract and history is first hypothesized. Then possible bidding decision criteria are discussed which leads us to adopt the bidding objective of maximization of expected income as the optimum, or near optimum, solution to our bidding problem. A detailed discussion of cost evaluation and competitors' behavior provides the basis from which this optimum, or

near optimum, bid can be obtained.

In Chapter IV, the paper is summarized and a concluding discussion is made of the applicability of a closed competitive bidding model.

In the Appendix, we follow through a simple, hypothetical bidding problem.

CHAPTER II

REVIEW OF THE CLOSED COMPETITIVE BIDDING LITERATURE

In this chapter, we review the closed competitive bidding literature that is relevant to the bidding situations discussed in Chapter III. An examination of this literature readily reveals the apparent inadequateacy of creditable research at the present time. In fact, the majority of the writings are simplified versions of Friedman's original investigation. A notable exception is Broemser's dissertation.

Friedman's Analysis

The first mention in the operations research literature of a closed competitive bidding model is by Friedman[1] in 1956. He shows the value of a bid to be the probability of winning the competition with a given bid times the income to be received if the competition is won with that bid. That is, he introduces the concept of expected income to competitive bidding. He uses the maximization of this function as his decision criterion.

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[1]Lawrence Friedman, "A Competitive-Bidding Strategy," Operations Research, Vol. 4 (No. 1, February, 1956), pp. 104-112.

Friedman employs the previous bidding patterns of the competitors to determine the probability of winning the competition with a given bid. If there is more than one other competitor and if these competitors submit independent bids, then the probability of winning the competition with a given bid is the product of the probabilities of winning against each competitor. If a competitor's bidding strategy cannot be characterized from historical bidding information, or if the identity of the competitors is unknown, the notion of the "average" competitor is utilized.

For the simultaneous bidding situation, Friedman suggests a simple "backing-off" procedure. That is, one first finds the bid that maximizes the expected income for each bidding opportunity. These are the "optimum" bids. However, if any resource constraints are exceeded, each "optimum" bid is decreased by some given amount and the resulting effect on the expected income is noted. The new bid ("optimum" bid minus some given amount) that causes the minimum decrease in the expected income then becomes the new "optimum" bid and all other "optimum" bids revert to their previous levels. This procedure is repeated until no resource constraints are exceeded.

Many of the concepts employed by Friedman are incorporated into the closed competitive bidding model developed in Chapter III.

The first reported successful application of a sophisticated closed competitive bidding model is made by Christenson.[2] He considers the situation of investment bankers bidding for new issues of corporate debt securities. He is able to use Friedman's model for the case where the identity and bidding strategy of all competitors is known since the corporate bond underwriting business is well established and highly concentrated in a few firms.

Edelman[3] also develops and demonstrates the merit of a closed competitive bidding model employing Friedman's analysis. Although Edelman provides a very limited discussion of the theoretical justification for his approach, his work is worth noting because of the outlines he presents for evaluating the costs associated with a bidding project and testing the sensitivity of the "optimum" bid to changes

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[2]Charles Christenson, Strategic Aspects of Competitive Bidding for Corporate Securities (Boston: Division of Research, Graduate School of Business Administration, Harvard University, 1965).

[3]Franz Edelman, "Art and Science of Competitive Bidding," Harvard Business Review, Vol. 43 (No. 4, July-August, 1965), pp. 53-66.

in the input information.

Arps[4] considers a range of various techniques for appraising a tract of undeveloped oil acreage: starting from the "best guess" estimate of all parameters and ending with the treatment of several parameters as indeterminate variables. Then, employing a variation of Friedman's analysis, Arps solves the closed competitive bidding problem with the objective of acquiring the maximum number of profitable tracts of acreage with a given amount of commitment (bidding) capital.

Other authors have followed Friedman's original analysis but have done little in the way of suggesting major innovations. See, for instance, Sasieni, Yaspan and Friedman; Miller and Starr; Park; and Ackoff and Sasieni.[5]

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[4]John J. Arps, "A Strategy for Sealed Bidding," 1965 Symposium on Petroleum Economics and Evaluation (Dallas, 1965), pp. 28-36.

[5]Maurice Sasieni, Arthur Yaspan, and Lawrence Friedman, Operations Research - Methods and Problems (New York: John Wiley and Sons, Inc., 1959); David W. Miller and Martin K. Starr, Executive Decisions and Operations Research (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1960); William R. Park, "How Low to Bid to Get Both Job and Profit," Engineering News-Record, Vol. 168 (No. 16, April 19, 1962), pp. 38-40; Russell L. Ackoff and Maurice W. Sasieni, Fundamentals of Operations Research (New York: John Wiley and Sons, Inc., 1968).

Broemser's Analysis

Broemser[6] develops and tests two closed competitive bidding models: one a bidding "optimization" model for the single bidding situation that maximizes the expected income of the bid, and another bidding "optimization" model for the sequential bidding situation that maximizes the long run expected discounted present value of the bid subject to capacity constraints.

Broemser finds that bidding does not become more aggressive as the number of competitors increases. Therefore, he rejects Friedman's independence assumption which implies that the winning bid does depend on the number of competitors. That is, Friedman's analysis predicts that as the number of competitors increases, the lower the probability of a given competitor winning the competition and, thus, the more aggressively that competitor is likely to bid. (However, it would seem more reasonable for Broemser to conclude that competitors do not base their bids on their estimates of other competitors' bids rather than reject the independence assumption). To avoid having to assume independence,

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[6] Gary Milton Broemser, "Competitive Bidding in the Construction Industry" (Unpublished Ph.D. dissertation, Stanford University, 1968).

Broemser considers only the lowest, or winning, competitor's bid. Then he constructs a single bidding situation model based on Friedman's analysis, but with the above mentioned alteration. This model is tested and found to be superior to an intuitive or non-systematic manner of bidding.

The construction industry has two features that enable Broemser to develop a sequential bidding model. First, it is the usual practice in the United States of public and many private owners to require "bidding bonds" which state a contractor's dollar bidding capacity. Second, there is usually a standard progress payment schedule which partly defines a project's cash flows. This model is tested and found to be superior to his single bidding situation model discussed above.

Vickrey

Vickrey[7] examines bidding from another viewpoint: the "optimum" allocation of resources within the economy. He points out that the "Dutch" auction,[8] and hence the

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[7] William Vickrey, "Counterspeculation, Auctions, and Competitive Sealed Tenders," The Journal of Finance, Vol. 16 (No. 1, March, 1961), pp. 8-37.

[8] In a "Dutch" auction, the auctioneer or judge announces prices in an ascending sequence. The first and only bid is the winner and concludes that transaction.

method of bidding that is being considered in this paper,[9] is in many instances non-Pareto optimal[10] in its allocation of resources. This is true because, in determining their bid, a competitor should not only consider the value of the prize to themselves, but also what their competitors' bidding strategies could be. To be a Pareto optimal allocation of resources, each competitor would have to bid just the amount that the prize is worth to them, taking no consideration of what anyone else will do.

Vickrey shows that the optimum allocation of resources can be assured if the required bidding procedure is altered to request bids on the understanding that the prize will be made to the lowest bidder, but on the basis of the price or bid amount set by the second lowest bidder. If this procedure is followed, then the optimum strategy (assuming

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[9]Both the "Dutch" auction and closed competitive bidding have the same factors of uncertainty: the value of the prize and the competitors' bids.

[10]C. E. Ferguson, <u>Microeconomic Theory</u> (Homewood, Illinois: Richard D. Irwin, Inc., 1966), p. 385; defines "... any organization (point) is said to be Pareto optimal or Pareto efficient when every reorganization that augments the value of one variable necessarily reduces the value of another".

the absence of collusion among the competitors) for each competitor will be to set their bid equal to the full value of the prize to themselves; that is, the lowest amount they could do the job (contract) for without incurring a loss, or the amount at which they would be on the margin of indifference as to whether they obtain the prize or not.

Bidding more than this full value would then only diminish their chances of winning at what would have been a profitable, or, at least, not unprofitable, bid price and would not affect the amount this competitor would actually receive if they were the successful bidder. Bidding less than full value, on the other hand, would increase their chances of winning, but only under circumstances that would involve them in an unprofitable transaction.

Although this method of awarding bidding contracts may be "optimum," it is not the practice that is commonly followed in bidding situations. Hence, in this paper, a prize is assumed to be awarded by the usual procedure: to the lowest bidder at their bid price.

CHAPTER III

A GENERAL CLOSED COMPETITIVE BIDDING MODEL

In this chapter, we develop a general closed competitive bidding model. Assuming that we have previously decided to submit a bid for a specified contract and employing this model, we discuss the determination of the bid that maximizes the expected income from this contract.

Bidding Contract and History

Company 1 competes against N-1 other companies: competitors 2,---,n,---,N.[1] A portion of the operations in this industry is allocated by closed competitive bidding. This is here defined to mean that each company may submit one independent, sealed bid for some indivisible object; for example, a contract of some sort. All bids are due in by the specified time and this bidding is only done once. The company who submits the lowest bid is declared the winner, and the contract is awarded to them for the

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[1] n is used to denote the general competitor; that is, $2 \leq n \leq N$.

fixed price that they bid.[2]

Contract M is proclaimed open for the submission of closed competitive bids. It will be awarded in the above defined manner at time t_M. Contracts 1,---,m,---,M-1 have previously been awarded by closed competitive bidding at times t_1,---,t_m,---,t_{M-1} in the past.[3]

Decision Criterion

The first step in submitting a closed competitive bid is for a company to adopt some bidding strategy or decision criterion which reflects that company's objectives or goals. Intuitively this would seem to be to win the bidding contract, but with a bid that assures some acceptable level of income.

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[2]This is then a lump sum bidding contract as differentiated from other forms. Here the contractor (winning bidder) agrees to fulfill the specifications of the contract for a fixed price (bid), and receives this price, but, in turn, must pay out whatever it costs them to do the work. A unit price bidding contract is based on a specified price (bid) per unit of work. A cost plus bidding contract can be either of a fixed fee or incentive fee type. With the fixed fee, the contractor receives payment for whatever it costs them to do the work plus a fixed fee (bid) as their income. With the incentive fee, the contractor is paid their actual costs to do the work plus (or minus) some incentive fee (bid).

[3]$1 \leq m \leq M-1 < M$ and, correspondingly,

$t_1 \leq t_m \leq t_{M-1} < t_M$.

As we show later in this chapter, there are three stages of analysis involved in the preparation of a bid. First, there must be an evaluation of the (estimated) costs of fulfilling the requirements of the contract. Second, there must be an assessment of the (possible) bidding behavior of the competitors. Finally, the results of the first two stages are combined. That is, the costs of fulfilling the requirements of the contract imply the income to be derived from a given bid if this bid is successful, and the bidding behavior of the competitors defines whether that bid will be successful or not.

Therefore, it seems clear that an abnormally high bid, while leading to a very profitable payoff, is also accompanied by a relatively low success probability. On the other hand, an abnormally low bid may virtually ensure success, but leads to an unsatisfactory level of payoff. Both ends of the scale thus lead to a low income "expectation"; that is the product of the payoff and the probability of realizing this payoff.

Generally, somewhere within this range there is a particular bid which generates the optimum trade-off between profitability and success probability. At this optimum price, the income expectation is at a maximum.

This implies that any price above the optimum causes a reduction in the success probability which more than counterbalances the effect of additional income. Conversely, any price below the optimum produces a reduction in the income which outweighs the benefits achieved from the higher success probability. Figure III.1 depicts the above described relationships.

There are a number of possible bidding objectives that a company could adopt; for example:

a. maximize their expected income;

b. minimize their expected losses;

c. minimize the income of their competitors;

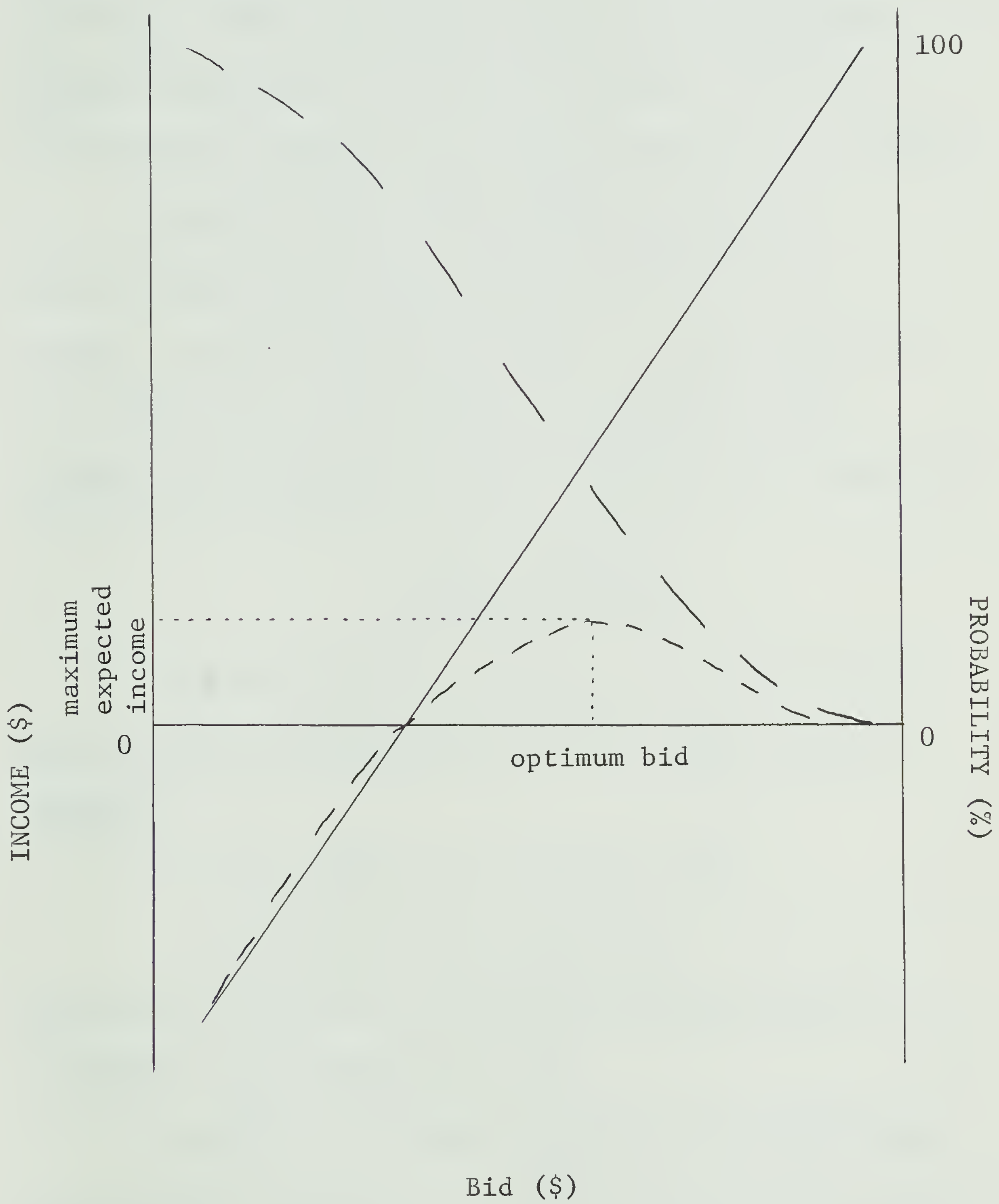
d. maximize the losses of their competitors; or

e. gain a satisfactory return on invested capital.

These, a combination of these, or other objectives could be adopted and each could lead to a different solution when applied to a bidding problem.

However, as we have previously discussed, the essential conflict in the bidding process is the trade-off caused by a bid between the income that results if the bidding is won with this bid, and the probability of winning the bidding with that bid. Therefore, it seems reasonable to designate expected income as the basis of

FIG. III.1--EXPECTED INCOME RELATIONSHIPS



——— income

— — probability of income

-------- expected income

our bidding objective, and the maximization of expected income as our decision criterion. This is certainly one of the more common approaches taken in the operations research literature to the bidding problem, and one of the simplest to manage in a bidding situation of our type.

The objective of this paper is to develop a general closed competitive bidding model from which company 1 could compute an optimum, or near optimum, bid for contract M. This is taken to be the bid which maximizes the expected income to company 1 from contract M.

Decision Function

From the preceding analysis, it appears that two major areas of uncertainties exist in closed competitive bidding:

1. the value of the contract; and
2. the bids of the competitors.

These form the basis of the decision function that is considered when determining a bid.

Let B_1^M be a bid that company 1 is contemplating submitting for contract M. If C_1^M is the cost that company 1 would incur on contract M, the income, I_1^M, that company 1 would receive if they won the bidding would be:

$$I_1^M = B_1^M - C_1^M.$$

However, company 1 will only win if they submit the lowest bid. That is, if competitor n bids B_n^M for contract M, company 1 will win if:

$$B_1^M < \min.(B_2^M,---,B_n^M,---,B_N^M).$$[4]

The decision function can then be expressed as:

$$B_1^M = \underline{fn}(C_1^M; \{B_n^M\}_{n=2}^N).$$

The above relationship provides the basis by which company 1 utilizes the information so as to set B_1^M to maximize the expected income from their bid.

Cost and Income Determination

C_1^M was earlier defined to be the cost that company 1 would incur on contract M. However, this value will not be known to them with certainty:

1. unless they actually win the bidding; and

2. until they finally complete the contract.

Therefore, viewed at pre-bid time, C_1^M is a random variable with some unknown probability distribution. However, we

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[4] The probability of a tie with the lowest competitor's bid (bids of equal price) is assumed to be zero. This assumption is quite reasonable as bidding is generally done to the closest cent, and the probability of the two lowest bidders submitting identical, independent bids on a contract that is maybe worth thousands of dollars is very close to zero.

may be able to estimate this distribution.

C_1^M may be conceptualized as the cost of consuming some, or all, of company 1's production capacity over the time they would take to complete contract M. Let $CC_1^M(t)$ be company 1's cost at time t of consuming some amount of their production capacity on contract M. Assume that company 1 would start contract M at time t_1^M and be completed at time T_1^M.[5]

If $\alpha_1^M(t)$ is a factor applied by company 1 to discount contract M's cash flows from their production capacity consumption $[CC_1^M(t)]$ at time t to time t_M, then:

$$C_1^M = \sum_{t=t_1^M}^{T_1^M} [CC_1^M(t) \times \alpha_1^M(t)].$$

That is, C_1^M is the marginal discounted cost.[6]

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[5] $t_1^M \leq t \leq T_1^M$.

[6] Marginal cost implies three distinct stages of evaluation:

1. current operations and the results which would prevail if the bidding had not been requested at this time;
2. the condition that the bid is successful and the consequences of the resulting operations; and
3. the condition that the bid is unsuccessful and the consequences of the resulting operations.

Therefore, some of the costs to be considered are not necessarily costs in the "traditional" sense, but may also include "opportunity" costs.

$CC_1^M(t)$ is a random variable. In fact, it is the sum of a number of factors that go to make up production capacity, and each of these are themselves random variables. Also, t_1^M may or may not be a random variable, but T_1^M is. Therefore, solving the above equation is very difficult when one considers these values as random variables. Because of this, a technique such as Monte Carlo simulation would be reasonably employed.

Since company 1 has some technological and financial constraints or limitations, and/or wishes to "rationalize" their consumption of production capacity, the shape of $\{CC_1^M(t)\}_{t=t_1^M}^{T_1^M}$ depends on present and (estimated) future scheduling requirements and assignments. T_1^M could then be unique to each possible schedule. The scheduling requires some rule, as does the discount rate to be applied, and these are other decision problems which must be solved in the pre-bid analysis. Their resolution provides the dynamic or sequential aspect to the bidding problem.

Assume that this scheduling, discounting and simulating are completed, and a probability distribution, $f(C_1^M)$, for C_1^M has been estimated.[7] For our purpose in finding expected income, it is useful to use the expected value, $E(C_1^M)$, of C_1^M. This is then:

$$E(C_1^M) = \int_{C_1^M=-\infty}^{\infty} C_1^M \times f(C_1^M)\, dC_1^M.$$

The simplest bidding strategy for company 1 to adopt is a mark-up system; that is:

$$B_1^M = \underline{fn}(C_1^M).$$

Some mark-up, k_1^M (≥ 0), on cost could be applied over and above cost to attain some desired income level for company 1 on contract M. This mark-up would reflect risk and efficiencies such as contract size and duration, but would not account for competitors' bidding behavior. For example, considerations such as the following would be made when deciding on the mark-up factor:

a. A job that is large relative to a company's production capacity or of long duration may be risky and so demand a high mark-up.

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[7] See Richard M. Anderson "Handling Risk in Defence Contracting," Harvard Business Review, Vol. 47 (No. 4 July-August, 1969), pp. 90-98; for an illustration of cost as a random variable.

b. A job that is small relative to a company's production capacity or of short duration may be inefficient and so demand also a high mark-up.

Company 1's bidding strategy and bid could be:

$$B_1^M = E(C_1^M) + I_1^M = E(C_1^M) + [k_1^M \times E(C_1^M)]$$

$$= E(C_1^M) \times (1 + k_1^M).$$

However, as we discussed earlier, a bidder should not only consider what a contract's value is to themselves, but also what their competitor's bidding strategies could be. Thus, a contractor will not necessarily bid exactly that amount which is equal to the value to them of winning this contract, nor will the winner of that contract necessarily be the ones who places the lowest value on winning the contract. Therefore, it is only adviseable to employ a bidding strategy that does not describe the competitors' behavior, such as the mark-up system just discussed, when a company has either a complete deficiency of information regarding their competitors and their bidding behavior, or is confronted by some time constraint when preparing a bid.

Competitors' Bids Evaluation

A company's chances of being the lowest bidder on a contract vary in some inverse relationship to their bid. If they are unable to calculate their chances of success prior to submitting their bid, they may gain a significant competitive advantage.

Each competitor's bid, as viewed by company 1 at pre-bid time, is a random variable subject to some unknown probability distribution. However, company 1 may have sufficient information to estimate this distribution from their knowledge of the competitor's behavior. If this behavior was present in the past and will continue in the future, a company may use historical bidding information to estimate each competitor's bid:

a. explicitly by estimating the analysis each competitor adheres to; and/or

b. implicitly by using this data directly and assuming that the statistical distribution of the bids in the future will be the same as in the past.

It should be noted that only those competitors who might actually submit bids should be considered as part of the competitive environment. In this case, assume that

competitors 2,___,n,---,N are all active and may enter this bidding competition.

Calculating Competitors' Bids Explicitly

An assumption that a company can make is that each of their competitors is, or, at least, can be as rational as themselves. This company may not know what analysis their competitors do, in fact, follow when determining their bids. However, they may assume either that each of their competitors uses:

a. the same or a known different analysis as they do; or

b. a unknown different analysis but that this analysis results in approximately the same outcome as theirs does.[8]

If a company is able to estimate each competitor's decision function and the value of its decision variables, they are able to determine a probability distribution of each competitor's bid. Even if this kind of information exists, which is doubtful, this type of analysis contains some constraints. Consider, for example, the following case.

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[8]If this argument is extended to its logical conclusion, all bids should then be approximately the same if all bidders have equal technological and financial abilities and limitations.

Assume that company 1 and competitor 2 are the only possible bidders on a contract and each is fully conscious of this fact. Company 1's bid should be a function of their estimate of competitor 2's bid, but competitor 2's bid should be a function of their estimate of company 1's bid. Then company 1's bid should be a function of their estimate of competitor 2's estimate of company 1's bid.

This argument must be terminated at some arbitrary point so that it can be operationalized. For example, company 1 can make the assumption that competitor n does not consider their competitors' bids when determining their bid; that is:

$$B_n^M = \underline{fn}(C_n^M);$$

and, therefore, from an earlier argument:

$$B_n^M = E(C_n^M) \times (1 + k_n^M).$$

Company 1's estimates of $E(C_n^M)$ and k_n^M are random variables. If these can be appraised, then they will define the resulting probability distribution, $f(B_n^M)$, of the random variable B_n^M from use of the above equation.

Each possible competitor may not actually compete for a given contract. From past experience, observations of present conditions and operations (especially production

capacity consumption and limitations), attendance at bid "lettings", and so on, company 1 may be able to assign a "compete - not compete" probability, $P(e_n^M)$, to each competitor where:

$$e_n^M = \begin{cases} 0 & \text{if competitor n does not bid on contract M; and} \\ 1 & \text{if competitor n does bid on contract M.} \end{cases}$$

That is, company 1 estimates that for contract M the probability of competitor n:

a. submitting a bid to be $P(e_n^M=1)$; and

b. not submitting a bid to be $P(e_n^M=0)$ or $1 - P(e_n^M=1)$.

Earlier we stated that company 1 will win contract M if:

$$B_1^M < \min.(B_2^M, ---, B_n^M, ---, B_N^M);$$

and lose otherwise. Viewed at pre-bid time, company 1 can now calculate the probability that they will win. Letting $P(B_1^M < B_n^M)$ be the probability that a bid of B_1^M is less than B_n^M for contract M, then:

$$P(B_1^M < B_n^M) = \int_{B_n^M=B_1^M}^{\infty} f(B_n^M)\, dB_n^M.$$

However, competitor n will only bid B_n^M with probability $P(e_n^M=1)$. Company 1 will definitely win over competitor n with probability $1 - P(e_n^M=1)$ as competitor n is estimated

not to submit a bid these times. Thus:

$$P(B_1^M < B_n^M) = [1 - P(e_n^M=1)] + [P(e_n^M=1) \times$$

$$\int_{B_n^M=B_1^M}^{\infty} f(B_n^M)\, dB_n^M].$$

If $B_2^M, ---, B_n^M, ---, B_N^M$ are independent,[9] then:

$$P(B_1^M < \min.(B_2^M, ---, B_n^M, ---, B_N^M)) = \Pi_{n=2}^{N} P(B_1^M < B_n^M).$$

That is, company 1's probability of winning contract M with a bid of B_1^M is the product of the probabilities of this bid winning against each competitor. Thus, given independence:

$$P(B_1^M < \min.(B_2^M, ---, B_n^M, ---, B_N^M)) =$$

$$\Pi_{n=2}^{N} ([1 - P(e_n^M=1)] + [P(e_n^M=1) \times$$

$$\int_{B_n^M=B_1^M}^{\infty} f(B_n^M)\, dB_n^M]).$$

---

[9]This should hold true even though the bids of the competitors may be highly correlated with cost and mark-up.

Calculating Competitors' Bids Implicitly

Another method of evaluating the competitive environment is to use past bidding information directly. It is true that such factors as production capacity consumption may be unique to a particular competitor, for a particular contract and at a particular time. But, if the number of competitors for lapsed bidding contracts and the present one are large enough, and if the history of bidding information is long enough, this type of analysis has some validity, at least for an "average" contract and "average" competitor. This is the best approach in the absence of the additional information required to calculate competitors' bids explicitly as was just discussed.

It may be true that a competitor's bidding objectives and strategy change over time. If this is found to be the case, it may be necessary to "weight" the historical bidding information so as to place more emphasis on the most recent data.

All Bids Disclosed

One of two sets of historical bidding information is assumed to be available on contracts 1,---,m,---,M-1

at times $t_1, \text{---}, t_m, \text{---}, t_{M-1}$.[10] In the first case, the following data is known to company 1 regarding contract m at time t_m:

a. all the other competitors, 2,---,n,---,N, and the amounts that they bid, $\{B_n^m\}_{n=2}^N$;[11] and

b. their estimate of $E(C_1^m)$.[12]

Given this data and the condition that there has been a large number of contracts previously let, company 1 may have sufficient information to individually characterize each competitor's bidding strategy.[13]

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[10] In actual fact, there may not necessarily be one of two distinct sets of this data available, but a combination of them both. However, for simplicity in the analysis, this difference is assumed to exist.

[11] We assume here that there is and has been a stable set of competitors. That is, each competitor has bid on all past contracts, and no new competitor has entered this industry nor has any competitor left. This assumption is for notational expediency.

[12] Assume that company 1 has made a cost estimate for each past contract, but has not won any of these competitions. Once again, this assumption is for notational purposes.

[13] If this condition is not met, it may be possible to follow the analysis to be presented next: the case where only the winning bid is made public.

A distribution of the ratio $\{B_n^m \div E(C_1^m)\}_{m=1}^{M-1}$ can then be built-up.[14] Cost is used as the denominator of this ratio so as to relate, for projection purposes, a competitor's bidding strategy to some internal information that company 1 has available on a future contract. That is, this distribution can be used to define the probability distribution of B_n^M by multiplying each value of the ratio by $E(C_1^M)$.

We are here making the assumption that:

$$B_n^m = f\underline{n}(C_n^m);$$

where C_n^m and C_1^m are related. However, further research should be made to discover any other influences besides cost that could account for the different values of the ratio.

Now the analysis is to a point where there is sufficient information to calculate:

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[14]Although the assumption is made in footnote 12 that company 1 has not won any of the competitions, if, in fact, they have, then the "true" cost of those contracts that they won would be used:

a. as a check on their cost estimation procedures; and
b. in the ratio for the development of the distribution.

$$P(B_1^M < \min.(B_2^M,---,B_n^M,---,B_N^M)) =$$

$$\Pi_{n=2}^{N} ([1 - P(e_n^M=1)] + [P(e_n^M=1) \times$$

$$\int_{B_n^M=B_1^M}^{\infty} f(B_n^M)\, dB_n^M]).$$

That is, given independence, the above equation which was developed earlier defines the probability that company 1 wins contract M with a bid of B_1^M.

Only Winning Bids Disclosed

A company's chances of success or failure in a bidding situation depend only on the lowest competing bid. Therefore, the amount of the lowest competing bid provides a sufficient description of the pertinent competitive environment. Instead of characterizing each competitor's bidding strategy as was just discussed for the case in which all bids are announced, we now treat all the competitors together as if they are a single competitor. Then, instead of assuming that a competitor's bidding strategy can be inferred from historical bidding information, we now assume that the winning bidding strategy can be deduced from past bidding data. No longer is it necessary to relate a given bidder to their bid.

In this second case, the following information is known to company 1 regarding contract m at time t_m:

a. only the winning bidder and the amount they bid, $\min.B^m$;[15] and

b. $E(C_1^m)$.

Similar to the preceding case, a distribution of the ratio $\{\min.B^M \div E(C_1^m)\}_{m=1}^{M-1}$ can be built-up, and used to define the probability distribution, $f(\min.B^M)$, of $\min.B^M$ by multiplying each value of the ratio by $E(C_1^M)$. A prior assumption now becomes:

$$\min.B^m = \underline{f}n(\min.C^m);$$

where $\min.C^m$ is the winning bidder's estimate of their cost of fulfilling contract m, and $\min.C^m$ and C_1^m are related.

Two possibilities exist at this point of the analysis. The first of these, which, in fact, is the one that has been assumed to now, supposes that the probability of winning the competition depends on the number of competitors against whom a company is bidding.[16] The

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[15] Although the assumption was made for notational expediency (see footnote 12) in the preceding case that company 1 has not won any of the past contracts, if contracts that they have won are included in this case, they will be, in essence, competing against themselves.

[16] See Friedman, "A Competitive-Bidding Strategy," p. 109.

reasoning here is that the more ("rational") bidders who compete for a given job, the lower the chances are that any one of them will get the job. This would imply that the probability distribution of min.B^m depends on the number of competitors for that contract.

One method of adjusting the previous analysis to account for this possibility is to use $P(e_n^M=1)$ and $1 - P(e_n^M=1)$ to calculate P(1 competitor),---,P(n competitor), ---,P(N-1 competitors). Separate distributions of the ratio [min.$B^M \div E(C_1^m)$] are built-up on those contracts for which there was estimated to be 1 competitor,---,n competitors,---, N-1 competitors. An aid in this part of the analysis may be to use Christenson's[17] observation that, in general, the number of bids submitted on an issue vary inversely with the size of that issue. In any case, some estimation must be attempted as only the winning bidder is publicly announced in this instance.

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[17] Christenson, Strategic Aspects of Competitive Bidding for Corporate Securities, p. 21.

Each of these distributions defines the probability distribution, $f(\min.B^M)|n$ competitors, of $\min.B^M|n$ competitors by multiplying each value of the ratio by $E(C_1^M)$. Then the probability that B_1^M is successful against a given number of competitors is the probability this bid is successful against this number of competitors times the probability that many competitors will compete. The probability that B_1^M is successful against any number of competitors is the sum of the probabilities that B_1^M is successful against a given number of competitors. Following an earlier analysis and given independence, we now have

$$P(B_1^M < \min.(B_2^M,\text{---},B_n^M,\text{---},B_N^M)) = P(B_1^M < \min.B^M)$$

$$= \sum_{n=1}^{N-1}\left(\left[\int_{\min.B^M=B_1^M}^{\infty} f(\min.B^M)|n \text{ competitors } d\min.B^M\right] \times\right.$$

$$\left.[P(n \text{ competitors})]\right).$$

The other possibility is to use Broemser's[18] conclusion that, from his data, the probability of winning the competition does not depend on the number of competitors bidding. Therefore, just the original, over-all distribution needs to be considered and now:

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[18] Broemser, "Competitive Bidding in the Construction Industry," p. 68.

$$P(B_1^M < \min.(B_2^M, ---, B_n^M, ---, B_N^M)) = P(B_1^M < \min.B^M)$$

$$= \int_{\min.B^M = B_1^M}^{\infty} f(\min.B^M)\ d\min.B^M.$$

In either case, tests should be made to find whether the independence assumption is valid for a specific bidding environment.

A Test of the Competitors' Bids

We have now shown a number of methods that can be employed to develop probability distributions of the competitors' bids. One pre-bid test of the validity of these methods may be to use a finding of Arps'.[19] He discovered by a detailed study of competitive sealed bidding over many years that the pattern of bids falling on a given tract (of land) appear to be susceptible to statistical treatment, and generally seem to follow a so-called "log-normal" type distribution.

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[19] Arps, "A Strategy for Sealed Bidding," p. 32.

<u>Bid Determination</u>

If company 1 loses the bidding for contract M with a bid of B_1^M, assume that:

$$I_1^M = 0.$$

In general, unsuccessful bidders obtain nothing; although in many cases of practical importance, such as those involving the submission of bids for large-scale contracts, the unsuccessful bidder may actually lose a considerable sum because of the costs involved in submitting the bid. One method of handling these submitting costs would be to include them in the contract costs, and attempt to develop some decision rule that would tell you whether to proceed with the estimating and bidding or not. Another method may be to ignore these costs; partly for purposes of simplicity, but, also, because they are "sunk" costs which are not affected by whether or not the company wins the competition.

If company 1 wins contract M with a bid of B_1^M, then:

$$I_1^M = B_1^M - C_1^M.$$

However, at pre-bid time, I_1^M is a random variable because C_1^M is. Therefore, we use the expected value,

$E(I_1^M)$, of I_1^M and the above equation becomes:

$$E(I_1^M) = B_1^M - E(C_1^M).$$

The expected income of a bid is the expected income if this bid is successful times the probability that bid is successful, minus the expected income if this bid is unsuccessful times the probability that bid is unsuccessful. For contract M, the expected income, $E(I_1^M)$, to company 1 of a bid of B_1^M is:

$$E(I_1^M|B_1^M) = ([E(I_1^M|B_1^M < \min.(B_2^M,---,B_n^M,---,B_N^M))]$$

$$\times [P(B_1^M < \min.(B_2^M,---,B_n^M,---,B_N^M))])$$

$$- ([E(I_1^M|B_1^M > \min.(B_2^M,---,B_n^M,---B_N^M))]$$

$$\times [P(B_1^M > \min.(B_2^M,---,B_n^M,---,B_N^M))])$$

$$=([E(I_1^M|B_1^M < \min.(B_2^M,---,B_n^M,---B_N^M))]$$

$$\times [P(B_1^M < \min.(B_2^M,---,B_n^M,---B_N^M))])$$

$$- ([0] \times [1 - P(B_1^M < \min.(B_2^M,---,B_n^M,---,B_N^M))]).$$

$$= E(I_1^M|B_1^M < \min.(B_2^M,---,B_n^M,---B_N^M))$$

$$\times P(B_1^M < \min.(B_2^M,---,B_n^M,---,B_N^M)).$$

Company 1 now has sufficient information to solve the above equation with a given B_1^M. To maximize this relationship, a matrix or graph of different B_1^M and their resulting $E(I_1^M)$'s is computed. From this, $\max_{B_1^M} E(I_1^M)$ should be readily visable. This is company 1's optimum, or near optimum, bid for contract M as it maximizes the expected income from contract M.

See the Appendix for an example solution to a simple, hypothetical bidding problem.

CHAPTER IV

SUMMARY AND CONCLUSIONS

An indication of the importance to the economy of closed competitive bidding is provided by Broemser and Arps. Broemser[1] estimates that over $40 billion worth of construction contracts were expected to be awarded in the United States in 1968 by this method. Arps[2] relates that the March, 1962 sale by closed competitive bidding of offshore oil and gas leases in the Gulf of Mexico committed some $920 million in bids by the petroleum industry. This amounts to approximately 20% of that industries total exploration expenditures for that year. From these two reports, it appears that closed competitive bidding plays an important role in the allocation of an economy's resources, at least very significantly in some industries.

But, is it important for a company to undertake research on bidding? Again Arps[3] provides an answer when

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[1] Broemser, "Competitive Bidding in the Construction Industry," p. 59.

[2] Arps, "A Strategy for Sealed Bidding," p. 28.

[3] Arps, "A Strategy for Sealed Bidding," p. 28.

he states that a study of the amounts "left on the table" strongly suggests that the actual choice of a number (bid price) in many cases is based more on hunch or intuition than on a systematic study of the subject. Also, actual tests by Edelman and Broemser,[4] and a survey by Paranka[5] prove the validity of models when applied to closed competitive bidding situations.

Our model developed and discussed in Chapter III shows a bid can be submitted based on:

1. cost; or
2. cost and competition's bids by calculating these bids:
    a. explicitly; or
    b. implicitly when:
        i. all bids on past contracts are disclosed; or
        ii. only the winning bids on past contracts are disclosed.

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[4]Edelman, "Art and Science of Competitive Bidding," pp. 53-66; Broemser, "Competitive Bidding in the Construction Industry."

[5]Stephen Paranka, "The Pay-Off Concept in Competitive Bidding," Business Horizons, Vol. 12 (No. 4, August, 1969), pp. 77-81.

Which basis of these outlined should a company follow? There appears to be no general correct answer to this question. This would depend on the specific bidding situation and the information that a company has available. Therefore, one of the first steps in the application of such a closed competitive bidding model is the systematic gathering of relevant information. Only then can a specific bidding model be built, tested and applied.

This paper has attempted to outline the more important dimensions of a general closed competitive bidding problem. It is hoped that once these are understood, the information that is available can be used to achieve the best results.

APPENDIX

In this appendix we follow through a simple closed competitive bidding problem based on a set of hypothetical historical information. Our analysis is for the case discussed in Chapter III when only the winning bid is disclosed. We determine the bid that maximizes the expected income from a given contract.

Bidding Information

Assume that company 1 is faced with a closed competitive bidding contract, 51, for which they estimate their expected cost, $E(C_1^{51})$, to be \$2100. Assume also that company 1 has the following bidding information on 50 past contracts, 1,---,m,---,50, that they did not win:

1. their estimate of the expected cost, $E(C_1^m)$; and
2. the winning bid, $\min.B^m$.

Bid Determination

The first step in determining the bid that maximizes the expected income for contract 51 from the above information is to calculate the ratio min.$B^m \div E(C_1^m)$ for each historical contract. Table A.1 displays the past bidding data and the above calculation.

Next the ratios are grouped into a frequency distribution which, in turn, is used to derive a cumulative probability distribution of a value being less than the ratios. For example, the lowest ratio is .85 and it occurs once. Therefore, there is a probability of 1.00 that .845[1] is less than all the ratios, and a probability of .98 [1.00 - (1 $\div$ 50)] that .855 is less than all the ratios. The cumulative probability is used to determine the probability of winning contract 51. Figure A.1 presents the distributions of frequency and cumulative probability.

Table A.2 summarizes the analysis for the determination of the optimum bid. First a bid, B_1^{51}, is calculated. The expected cost, $E(C_1^{51})$, is subtracted from B_1^{51} to yield the expected income, $E(I_1^{51})*$, if B_1^{51} is successful. This result

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[1].845 is arbitrarily taken instead of .85 to avoid the possibility of a tie between bids.

TABLE A.1--HISTORICAL BIDDING INFORMATION

Contract m	$E(C_1^m)$	min.B^m	min.B^m ÷ $E(C_1^m)$	Contract m	$E(C_1^m)$	min.B^m	min.B^m ÷ $E(C_1^m)$
1	$3200	$3232	1.01	26	$1100	$1001	.91
2	400	400	1.00	27	1100	1001	.91
3	300	273	.91	28	2500	2725	1.09
4	1000	1010	1.01	29	1700	1751	1.03
5	1300	1300	1.00	30	900	954	1.06
6	700	714	1.02	31	500	495	.99
7	1400	1358	.97	32	800	888	1.11
8	1200	1020	.85	33	600	558	.93
9	200	202	1.01	34	200	192	.96
10	100	103	1.03	35	3400	3400	1.00
11	300	288	.96	36	2300	2369	1.03
12	2100	2142	1.02	37	1100	1045	.95
13	700	721	1.03	38	1800	1836	1.02
14	1400	1316	.94	39	1300	1339	1.03
15	1400	1344	.96	40	3700	3552	.96
16	3100	3162	1.02	41	800	848	1.06
17	800	824	1.03	42	1600	1584	.99
18	600	612	1.02	43	200	200	1.00
19	2400	2376	.99	44	200	204	1.02
20	4200	4497	1.07	45	100	111	1.11
21	5300	5459	1.03	46	200	200	1.00
22	1600	1456	.91	47	3900	3783	.97
23	100	89	.89	48	400	388	.97
24	1200	1248	1.04	49	500	435	.87
25	100	97	.97	50	1700	1547	.91

FIG. A.1--FREQUENCY AND CUMULATIVE PROBABILITY FROM HISTORICAL BIDDING INFORMATION

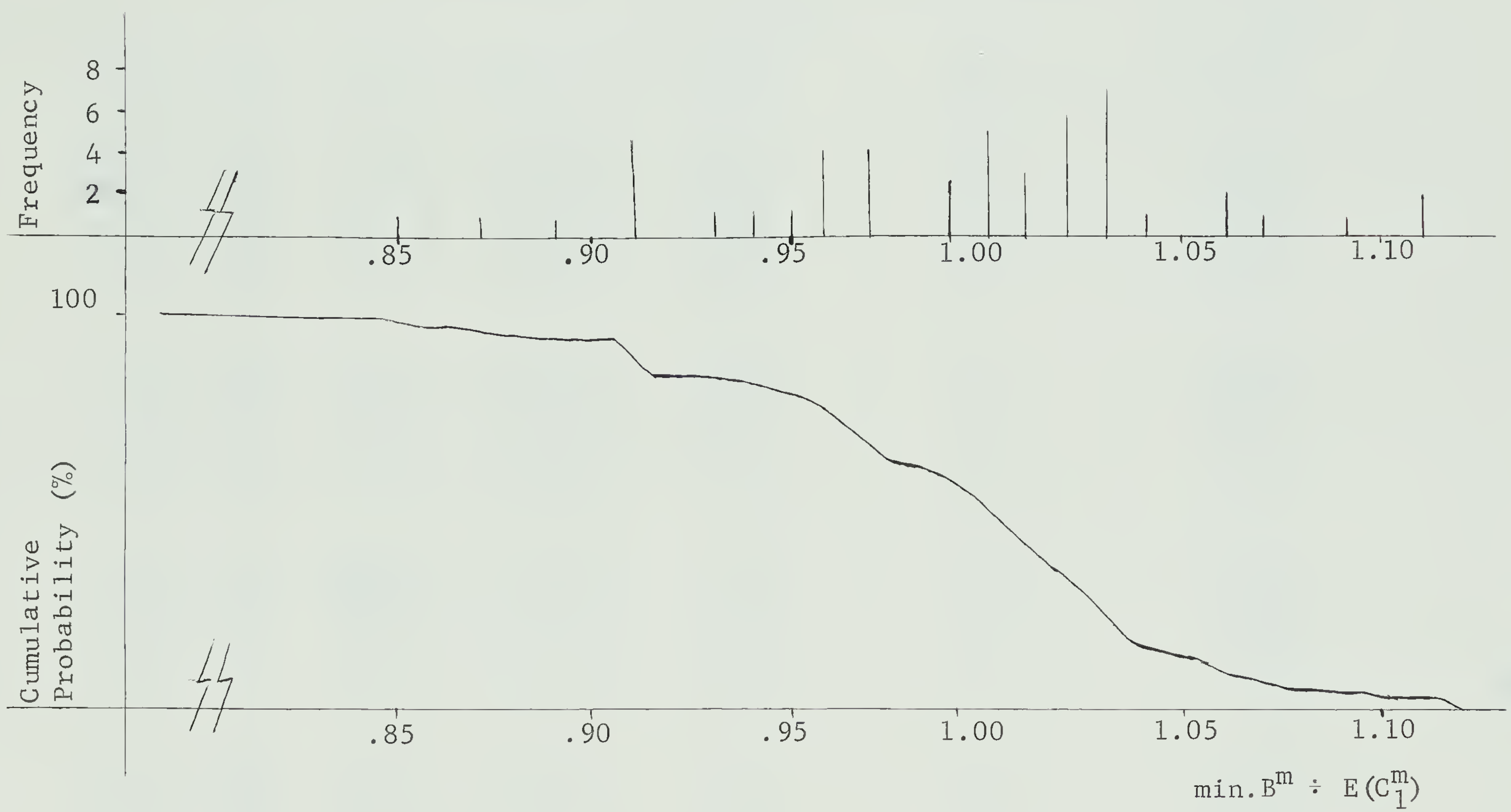


TABLE A.2--BID DETERMINATION

(1) B_1^{51}	(2) $E(C_1^{51})$	(3) $E(I_1^{51})^*$ [(1) - (2)]	(4) $P(B_1^{51} < \min. B^{51})$	(5) $E(I_1^{51})$ [(3) x (4)]
.845 x 2100 = $1774.50	$2100.00	$-325.50	1.00	$-325.50
.855 x 2100 = 1795.50	2100.00	-304.50	.98	-298.41
.865 x 2100 = 1816.50	2100.00	-283.50	.98	-277.83
.875 x 2100 = 1837.50	2100.00	-262.50	.96	-252.00
.885 x 2100 = 1858.50	2100.00	-241.50	.96	-231.84
.895 x 2100 = 1879.50	2100.00	-220.50	.94	-207.27
.905 x 2100 = 1900.50	2100.00	-199.50	.94	-187.53
.915 x 2100 = 1921.50	2100.00	-178.50	.84	-149.94
.925 x 2100 = 1942.50	2100.00	-157.50	.84	-132.30
.935 x 2100 = 1963.50	2100.00	-136.50	.82	-111.93
.945 x 2100 = 1984.50	2100.00	-115.50	.80	- 92.40
.955 x 2100 = 2005.50	2100.00	- 94.50	.78	- 73.71
.965 x 2100 = 2026.50	2100.00	- 73.50	.70	- 51.45
.975 x 2100 = 2047.50	2100.00	- 52.50	.62	- 32.55
.985 x 2100 = 2068.50	2100.00	- 31.50	.62	- 19.53
.995 x 2100 = 2089.50	2100.00	- 10.50	.56	- 5.88
1.005 x 2100 = 2110.50	2100.00	+ 10.50	.46	+ 4.83
1.015 x 2100 = 2131.50	2100.00	+ 31.50	.40	+ 12.60
1.025 x 2100 = 2152.50	2100.00	+ 52.50	.28	+ 14.70
1.035 x 2100 = 2173.50	2100.00	+ 73.50	.14	+ 10.29
1.045 x 2100 = 2194.50	2100.00	+ 94.50	.12	+ 11.34

[illegible]

TABLE A.2 (continued)

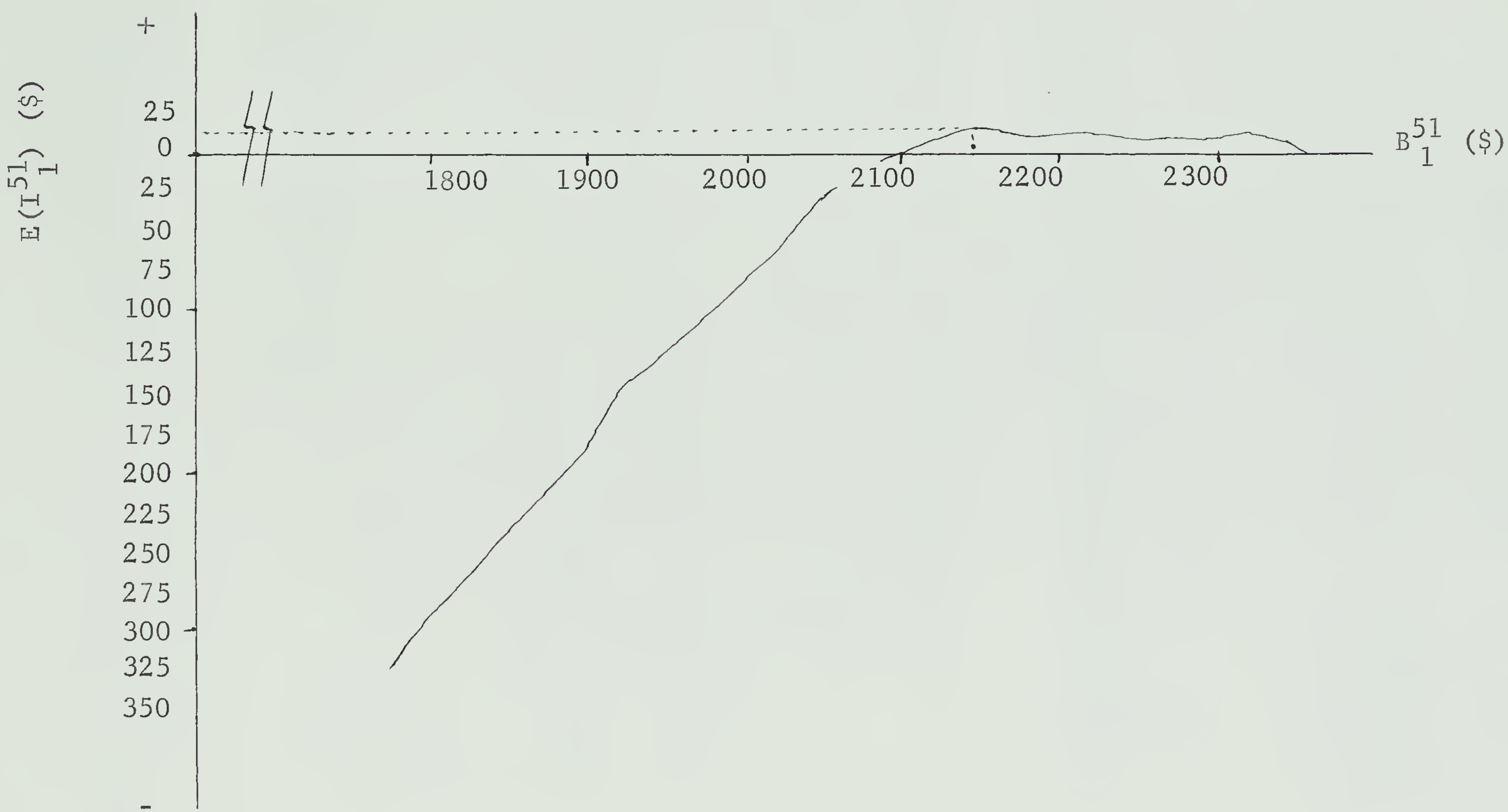
1.055 x 2100 =	2215.50	2100.00	+115.50	.12	+ 13.86
1.065 x 2100 =	2236.50	2100.00	+136.50	.08	+ 10.92
1.075 x 2100 =	2257.50	2100.00	+157.50	.06	+ 9.45
1.085 x 2100 =	2278.50	2100.00	+178.50	.06	+ 10.71
1.095 x 2100 =	2299.50	2100.00	+199.50	.04	+ 7.98
1.105 x 2100 =	2320.50	2100.00	+220.50	.04	+ 8.82
1.115 x 2100 =	2341.50	2100.00	+241.50	.00	0

is multiplied by the probability B_1^{51} is successful, $P(B_1^{51} < \min. B^{51})$, to give the expected income $E(I_1^{51})$.

From the table, we see that the maximum expected income is \$14.70 from a bid of \$2152.50 with a probability of success of .28. This is company 1's optimum bid. Figure A.2 depicts expected income from different bids.

One thing that should be noted regarding this particular problem is the possibility of company 1 bidding some other amount; for example, \$2215.50. This is \$63.00 above the optimum bid and has a probability of success of .12 for an expected income of \$13.86. It so happens in this case that there is little difference, \$.84, between this expected income and the maximum expected income. For this type of situation, a company may not necessarily restrict their bidding strategy to maximizing expected income, but adopt some other strategy depending on risks and their utility function.

FIG. A.2--EXPECTED INCOME



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